

new arsenic compounds. Short chapters are devoted to the therapeutics of tumours, inflammation, blood-diseases, gout, fever, and disturbances of the circulation and digestion so far as these have been determined by experimental methods. It is, of course, impossible to treat this extensive programme exhaustively in 174 pages, and the author seems rather to have aimed at giving a general idea of what is being done to advance therapeutics experimentally with the object of arousing the interest of the students and younger practitioners of medicine in the subject. The book seems well fitted to attain this object, for it is written in an easy style, and deals with some of the most interesting topics in medicine at the present time. On the other hand, the chapters are very unequally written. In some instances pages are devoted to detailed description of surgical methods (pp. 34-36) or of individual experiments, which seem out of place in an introductory handbook, while other subjects are treated too briefly for anyone to follow except the expert; and there is very little attempt made to differentiate the fundamental experiment from the less important or less generally accepted result.

The author tends too often to leave the solid ground for speculations which are often based on experiments which, to say the least, have not yet received general assent. In a book primarily designed for German medical students, perhaps it is right to direct their attention chiefly to authors of their own nationality whose works they can read, but we cannot help thinking that some of the chapters would have been improved by wider reading. For example, the chapter on vaccines might have been rendered more intelligible and also more up-to-date.

The book is not free from serious errors; for example, where (p. 11) it is stated that Hunt found alcohol protects mice against the nitriles; and the antidotal effect of sodium sulphate in barium poisoning is surely due to the barium being precipitated, and not to the restoration of the sodium, as the author supposes (p. 13).

PROJECTIVE GEOMETRY.

Projective Geometry. By Prof. O. Veblen and Prof. J. W. Young. Vol. i. Pp. x+342. (Boston and London: Ginn and Co., 1910.) Price 15s. net.

IN the first page of their introduction the authors say:

"The starting-point of any strictly logical treatment of geometry must be a set of undefined elements and relations, and a set of unproved propositions involving them; and from these all other propositions (theorems) are to be derived by the methods of formal logic."

Here, in a nutshell, is the modern mathematician's creed; and it is significant that it should thus appear in a treatise on projective geometry, which at first sight would seem to be one of the most intuitive of the branches of mathematics.

In accordance with the above dictum, the authors give a brief discussion of the axioms of geometry so far as they are required for the purposes of this volume, rightly, we think, deferring the more com-

plete theory of order and continuity to a later stage. Enough, however, is done to make the reader aware of the numerous tacit, and often complex, assumptions made in the ordinary treatment of the subject. For instance, we have an explicit statement of the fundamental postulate:

"If A, B, C are points not all in a line, and D, E are distinct points such that (B, C, D), (C, E, A) are respectively collinear, then there is a point F such that (A, B, F) and (D, E, F) are respectively collinear."

With the help of this and a few other assumptions, a plane is defined in such a way that it can be *proved* that if A, B are any two points in a plane, every point of the line AB is in the plane. No one can fail to see that this is an improvement on the Euclidean definition of a plane, which is a question-begging assumption, based no doubt on the practical tests applied by masons and carpenters.

After this the reader is introduced to the fundamental operations of projection and section, and to the principle of duality. The latter is very properly stated, at the outset, with reference to three-dimensional space: that is, point and plane are correlative terms, not point and line. It is easy enough to deduce the special laws of duality for two-dimensional fields; and the more general form of statement at once brings home to the student the fact that, as a rule, the propositions of projective geometry arrange themselves in sets of four, only one of which need be formally proved. For instance, Pascal's theorem for a conic in a plane leads at once to Brianchon's theorem, and two corresponding theorems for a quadric cone.

Even yet it may be asserted that von Staudt is the great master of projective geometry, much as Gauss is the incomparable arithmetician. It is one of the great merits of this work that the influence of von Staudt's work is so apparent in it. For instance, involution is treated at a comparatively early stage; and this is important for several reasons. In the first place, it simplifies the proofs of many fundamental properties of conics; in the second, it shows the existence of a polar system, in a plane or in space, apart from the assumption of a quadric curve or surface defining it. Ultimately, of course, the best definition of a conic or quadric surface is that it is the locus of self-corresponding points in a polar system. This, with Staudt's theory of imaginary (or complex) elements, permits of the inclusion of "imaginary" conics and quadrics as actually existing things. It is to be hoped that the second volume will contain a sufficient account of Staudt's beautiful theory, which, as a rule, seems to be very imperfectly apprehended. As he unfolds it in the supplements to his "*Geometrie der Lage*," it is purely geometrical, though no doubt he was led to it by analysis—at least, this seems the most probable assumption.

Among the interesting points of the present volume there is a brief account of Staudt's theory of "throws" (*Würfe*), and his constructions for addition and multiplication. In the latter there is a slight modification, arising from a change in the order of deduction. What is here shown is that if we take any three points

on a line and provisionally label them $P(0)$, $P(1)$, (∞) , and if $P(x)$, $P(y)$ are provisional labels of any other two points, then there are projective constructions defining $P(x+y)$ and $P(xy)$, such that the laws for adding and multiplying labels obey the ordinary laws of algebra, e.g. $P(x+y)=P(y+x)$, including the limiting cases when x or y , or both, have one of the special values 0, 1, ∞ . Hence it follows that from the base points $P(0)$, $P(1)$, $P(\infty)$ we can construct a *rational scale* of points $P(r)$, where r is any rational, positive or negative number. It may be added that these are the only points on the line which can be reached from the base-points by projective constructions, and it would be a good exercise for the student to prove what is not absolutely demonstrated in the book, that this deduction is free from ambiguity; for instance, suppose $P(x)$, $P(y)$, $P(x+y)$ have each been deduced from the base-points by a chain of projective construction, it is required to prove that *the same point*, $P(x+y)$, is derivable from $P(x)$ and $P(y)$ by the single operation of addition.

The net result of these considerations may be put (among other ways) in the following form. Suppose we have a tetrahedral frame of lines with a point given on each line distinct from the two vertices of the tetrahedron which it contains. Then on each of the six lines we can construct a rational scale, and hence, by projective constructions alone, arrive at all points which can be defined by four rational homogeneous coordinates. This rational projective space is not continuous: to fill up the lacuna, it will be necessary to assume the existence of one linear continuum of points, and apparently this will be also sufficient.

Staudt, on the other hand, gives his constructions as the definitions of the addition and multiplication of throws; and because the laws of algebra are satisfied, he deduces the possibility of assigning numerical values to throws. On the whole, Staudt's procedure seems the more scientific, but it is not a matter of much importance.

A propos of involutions, attention may be directed to the proposition on p. 223: "Any projectivity in a one-dimensional form may be obtained as the product of two involutions." This is very interesting, because it shows that although involution is, in the first instance, a derivative idea (as a special case of projective correspondence) it may ultimately be regarded as elementary.

To indicate how far this volume proceeds, it will be sufficient to say that chapter x. deals with pairs and pencils of conics in a plane, and gives the typical algebraic forms according to the elementary divisors of the discriminant; while the next, and final, chapter is on families of lines, and treats briefly of ruled quadrics, line coordinates, and linear congruences and complexes. It might, by the way, interest applied mathematicians to point out that if we suppose a unit force acting along a given line, the six homogeneous coordinates of the line may be taken to be proportional to the moments of the force about the edges of the tetrahedron of reference.

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THE NEW PSYCHOLOGY.

Manual of Mental and Physical Tests: a Book of Directions compiled with special Reference to the Experimental Study of School Children in the Laboratory or Class-room. By Prof. G. M. Whipple. Pp. xix+534. (Baltimore, U.S.A.: Warwick and York, inc., 1910.)

IS psychology to rank among the exact sciences? This is the question which is at once raised when we look into Prof. Whipple's volume. We are reminded of Kant's famous pronouncement that psychology never could be a science, because it was impossible either to apply mathematics to its problems or to perform experiments upon the minds of others. Kant's dictum is a classical instance of the danger of prophesying the impossible. In the book before us the mathematical treatment of mental measurements is discussed in the third chapter, and the rest of the volume is made up of more or less happily devised experiments upon the minds and bodies of other people.

Yet nobody knows better than the author himself how relatively slight actual accomplishment has been. The contrast between the position of the psychologist and that of the doctor, for example, is very great. An insurance company will decide quite serious financial questions on the report of a medical man whose tests give sufficiently good *average* results for their purpose. But the psychologist has not yet achieved a position of such confidence. He deals with far subtler problems, and it is no fault of his that we have not yet begun to consult him on the future of our children. He frankly confesses that he is not ready to render such positive service; and whilst on one hand we may fairly congratulate ourselves on the fact that a serious attempt is being made to arrive at a better understanding of mental phenomena, it would, on the other hand, in Dr. Whipple's words, be wrong to speak

"as if a science of mental tests had already been achieved. . . . To make such an assertion is surely misleading, for . . . there is, at the present time, scarcely a single mental list that can be applied unequivocally as a psychical measuring-rod."

There is no general agreement about procedure, and in many cases psychologists do not know exactly what it is they are measuring, such is the "astounding complexity, variety, and delicacy of form of our psychical nature."

It would, nevertheless, ill become professional students of the older sciences to speak in contemptuous terms of a younger brother who is so conscious of his own shortcomings. Psychology may have long to wait for its Newton, but in the meantime the effort to collect facts in a scientific way should surely meet with every possible encouragement. The "man in the street" has long recognised the existence of differences in the mental characteristics of his friends. He accepts them in much the same spirit as folk regarded the weather, until the meteorologist began that painstaking collection of data which is just beginning to bear scientific fruit. We have more words to describe these personal differences than we have for